What College Students Need to Know about Decimals

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NO!

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Mathematically:

Fractions use the problem itself $\dfrac{a}{b}$ as a representation of an exact number. Eg. solution of $3x = 2$ is $\frac{2}{3}$

Decimals give an approximation of the solution on the number line.
Eg. solution of $3x = 2$ is 0.67 or 0.6667 or maybe 0.66666667

Historic and Cultural Difference

These approaches are also distinct historically and culturally

Historic: Fractions were invented between 1800 BC and 1600 BC in Egypt, and later played a prominent role in the western mathematics passed down from the Greeks. Decimals, as sexagesimal fractions (base 60), were invented at the same time by the Babylonians and came to western mathematics in the guise of Hindu base ten decimal numbers from the Arabs in the middle ages.

Cultural: Fractions are part of Pure, Exact Mathematics. Equations are True or False. Decimals are approximations motivated by mathematical applications. Equations may be only *approximately true.*

Decimals are Rounded

In normal use *a decimal represents any number that rounds to that number.*

For example:

0.5 could represent $0.45, 0.495, \frac{1}{2}, 0.504999,$ or 0.54999

$.50$ could represent $0.495, 0.4995, \frac{1}{2}, 0.504999$ but not 0.45

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Exact Mathematics: $1 + 2 = 3$ ALWAYS

Approximate Mathematics:

$.7 + 1.6 = 2.3$ rounds to $1. + 2. = 2.$

$1.4 + 2.3 = 3.7$ rounds to $1. + 2. = 4.$

(This is one example of why accurate approximation is important!)

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Exception to roundoff rule: Some calculators use fewer digits than normal to indicate an *exact* number, eg. 0.5 means $\frac{1}{2}$, and 0.025 means $\frac{1}{40}$. Money amounts eg. \$32.14 are usually considered *exact*.

Use Your Calculator!

Do not feel embarrased using your calculator to add, subtract, multiply and divide decimals. Decimals and calculators go together.

As recently as the 1940's a calculation like

$$2.34157921 + \frac{3.29531247}{1.69887375} = 4.27128336 \tag{1}$$

would be considered very time consuming and avoided if at all possible. People used mathematical methods which avoided this kind of calculation. However these methods were more complicated and less accurate.

Today calculation (1) is considered easy. We now teach simpler more accurate methods using calculations like (1). A calculator *must* be used.

In the "old days" we carried around many tables of special kinds of numbers such as logarithms. Now our scientific calculators have these built in with far more accuracy available.

High School No No's

Certain practices you probably learned in high school should not be used in college courses.

Repeating Fractions: You may have written $\frac{1}{3}$ as $0.\overline{3}$ or $\frac{4}{11}$ as $0.\overline{36}$ Don't do this in college. A precision must be specified for fractions and only very special fractions have short "repetends". For example find the repetend in

$$\frac{5}{19} = .263157894736842105263157894737$$

π is not 3.14: When doing calculations using π with a calculator do not use 3.14 or $\frac{22}{7}$. These are simply not accurate enough approximations. If you have a scientific calculator use the internal value of π. Otherwise use at least accuracy 3.14159

Don't write down intermediate results: When doing a problem arrange the calculations needed so that you can do the entire calculation in your calculator. Use $\boxed{(}$,$\boxed{)}$ or internal storage if you have it. Your calculator often is using digits not displayed. *Do not round until you have the final result.*

Using Calculator Order of Operations

In written mathematics there are standard conventions for order of operations, modern calculators such as $\mathrm{TI30XII}$, $\mathrm{TI83/4}$ adhere to these conventions.

1. Grouped expressions *such as ( ),[ ], fraction bar*
2. exponential expressions
3. multiplication and division
4. addition and subtraction

Two Common Mistakes

$$1.319 + \frac{3.412}{5.657} = 1.922 \text{ but } \frac{1.319 + 3.412}{5.657} = 0.836$$

$$\frac{9.847}{2.528} * 3.697 = 14.400 \quad \text{but} \quad \frac{9.847}{2.528 * 3.697} =$$

$$9.847 \div (2.528 * 3.697) = 9.847 \div 2.528 \div 3.697 = 1.054$$

Scientific Notation, floating point numbers

For very large or small numbers our usual decimal notation is awkward, for example 0.000000032157 or 357623899987.6. So we use *scientific* or *floating point* numbers. These are decimals $x, 1 \leq x < 10$ multiplied by a power of 10, eg./ $1.235 * 10^{12}$ or $9.237 * 10^{-8}$. A useful property of these numbers is that we can round the decimal part, even if the number is large.

Examples:

$134,729,684,371 \approx 1.3473 * 10^{11}$ $\quad$ $4.328 * 10^{7} \approx 43,280,000$

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The number of digits in the decimal part, including the digit to the left of the decimal point, is called the *number of significant digits*. Since calculators and computers do all their internal calculations using floating point numbers this is the best way to describe the accuracy of a decimal number even if it is not given in scientific notation. For example the following numbers are all given to 5 significant digits:

$4.3251,\ 8235700,\ 0.0032547,\ 23.217,\ 486.29,\ 2.3756 * 10^{7}$