# Combinations in Probability

First recall that $n!$ is the product of the first $n$ integers, eg. $4! = 4 * 3 * 2 * 1$. For the TI83/4 ! can be found on the `MATH PRB` menu. First enter $n$ then get !

The number of *combinations* of $n$ things $r$ at a time is denoted ${}_nC_r$. One starts with a set of $n$ items then forms the set of all possible subsets with $r$ elements. The number of elements in this set is ${}_nC_r$ For example if $S = \{a, b, c, d, e\}$ then $n = 5$ and if $r = 2$ we have the set

$$\big\{\{a,b\},\{a,c\},\{a,d\},\{a,e\},\{b,c\},\{b,d\},\{b,e\},\{c,d\},\{c,e\},\{d,e\}\big\}$$

So ${}_5C_2 = 10$. The formula is

$${}_nC_r = \frac{n!}{(n-r)!r!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots(1)}$$

Note the numerator and denominator have the same number of terms. Further all terms in the denominator can be cancelled so we always get an integer. For example

$${}_5C_2 = \frac{5!}{(5-2)!2!} = \frac{5*4*3*2*1}{(3*2*1)(2*1)} = \frac{5*4}{2*1} = 5*2 = 10$$

Another way of finding ${}_nC_r$ for small $n, r$ is to use *Pascal's Triangle* on the right. Each row represents one $n$ which is the second entry in the row. The numbers in the row are then ${}_nC_r$ starting from $r = 0$ on the left to $r = n$ on the right. The top entry is ${}_0C_0 = 1$ and the outside entries are ${}_nC_0 = {}_nC_n = 1$ and the other entries are the sum of the two above them. Entries in $n$th row add to $2^n$.

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

Of course the usual way to calculate ${}_nC_r$ is to use the ${}_nC_r$ key from the `MATH PRB` menu: first enter $n$ then get ${}_nC_r$ from the menu and then enter $r$. If you will be multiplying this number the entire ${}_nC_r$ should be in parentheses, eg. $(10\ {}_nC_r\ 2)$. Outside probability ${}_nC_r$ is written $\binom{n}{r}$.

**Application 1:** 10 marbles are in a can, 5 red 3 blue and 2 white. Two are picked together (no order). What is the probability of 2 blue? Of a red and a white? Answer: Think of the marbles of a given color as being numbered. Any pair is equally likely and the number of pairs is ${}_{10}C_2 = 45$ Since there are 3 blue marbles there are ${}_3C_2 = 3$ pairs of blue marbles, hence $P(2\text{ blue}) = 3/45 = 1/15 = 0.0667$ But there are $5 * 2 = 10$ pairs of one red and one white marble so $P(\text{red, white}) = 10/45 = 2/9 = 0.222$

**Application 2:** What is the probability of a flush (5 cards of the same suit) if a poker hand of 5 cards is dealt from a standard deck? Answer: There are ${}_{52}C_5 = 2598960$ possible poker hands. Each suit has 13 cards so there are ${}_{13}C_5 = 1287$ flushes of that suit, or $4 * 1287 = 5148$ possible flushes. Hence $P(\text{flush}) = 5148/2598960 = .00198$ or about 1 in 505 deals.