## 5.1.5 Special Case, Dynamic Plane Tessellations using two sided cells.

We can use our 3 dimensional transformation functions to show that every plane transformation function can be seen as a continuous action in space. We give a "simple" example which is not so simple in execution.

We use as our base cell a scalene right dynamic triangle.

In[ ]:=
```
Dtri[α_] := {{Blue, Polygon[α@{{0, 0, .01}, {1, 0, .01}, {0, 2, .01}}]},
  {Green, Polygon[α@{{0, 0, -.01}, {1, 0, -.01}, {0, 2, .01}}]
  }}
```

To picture it we use 3D graphics, but we must add a vertical measuring stick to give the scale

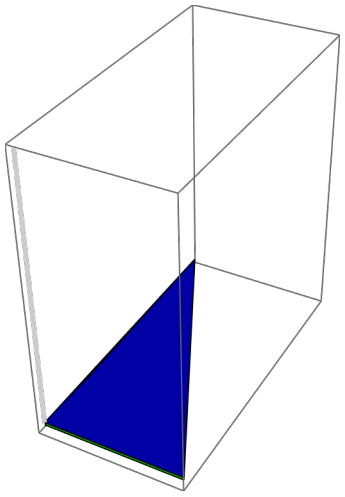
In[ ]:=
```
stick := {GrayLevel[.8], Thickness[.01], Line[{{0, 0, 0}, {0, 0, 2}}]}
```

Recall

In[ ]:=
```
ι3 := TranslationTransform [{0, 0, 0}]
```

In[ ]:=
```
Graphics3D [{stick, {Dtri[ι3]}}]
```

Out[ ]=

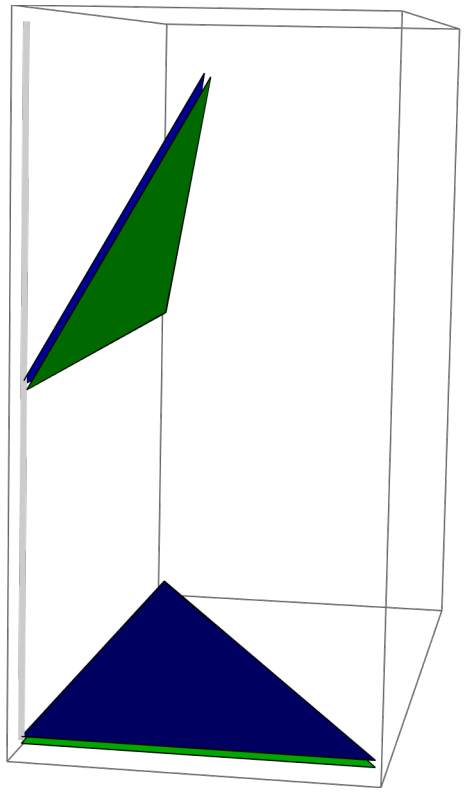


Superficially this looks like a plane triangle but note the green streak. A different view is

In[ ]:=
```
β := TranslationTransform [{0, 0, 1}]@* RotationTransform [-Pi / 3, {0, 1, 0}]
```

In[ ]:=
```
Graphics3D[{{GrayLevel[.8], Thickness[.01], Line[{{0, 0, 0}, {0, 0, 2}}]}, {Dtri[ı3], Dtri[β]}}]
```

Out[ ]=



Later we will make the measuring stick virtually invisible by changing the thickness and color .

In[ ]:=
```
stick2 := {GrayLevel[.99], Thickness[.001], Line[{{0, 0, 0}, {0, 0, 2}}]}
```

We can make a rotation of this triangle into a continuous space motion using

In[ ]:=
```
Clear[t]
```

In[ ]:=
```
ThreePointParabola[{q1_, q2_, q3_}, {t1_, t2_, t3_}] :=
 Module[{eq1, eq2, eq3, a1, a2, a3, b1, b2, b3, c1, c2, c3, sol},
  eq1 = {a1, a2, a3} t1^2 + {b1, b2, b3} t1 + {c1, c2, c3} == q1;
  eq2 = {a1, a2, a3} t2^2 + {b1, b2, b3} t2 + {c1, c2, c3} == q2;
  eq3 = {a1, a2, a3} t3^2 + {b1, b2, b3} t3 + {c1, c2, c3} == q3;
  sol = NSolveValues[eq1 && eq2 && eq3, {a1, a2, a3, b1, b2, b3, c1, c2, c3}][[1]];
 t^2 {sol[[1]], sol[[2]], sol[[3]]} + t {sol[[4]], sol[[5]], sol[[6]]} + {sol[[7]], sol[[8]], sol[[9]]}]
```

In[ ]:=

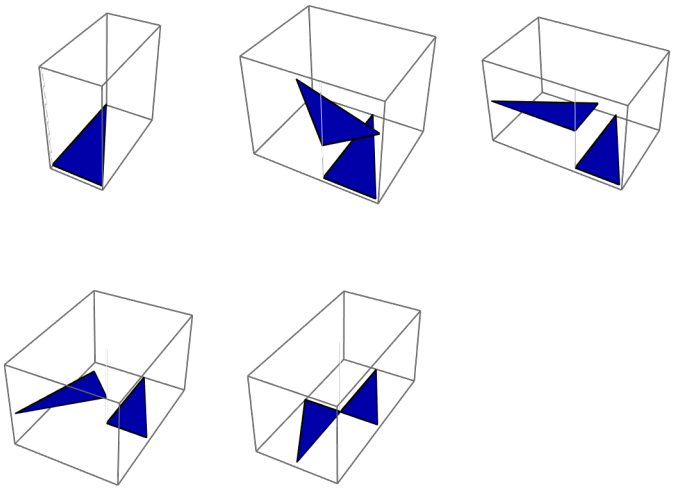
In[ ]:=
```
G[t_] = ThreePointParabola[{{0, 0, 0}, {0, 0, 1}, {0, 0, 0}}, {0, 1, 2}]
```

Out[ ]=
$$\{0.,\ 0.,\ 0. + 2.\ t - 1.\ t^2\}$$

In[ ]:=
```
Γ3[t_] := TranslationTransform[G[t]] @* RotationTransform[t Pi / 2, {0, 0, 1}]
```

In[ ]:=
```
GraphicsGrid [
 {{Graphics3D [{stick2 , Dtri[Γ3[0]]}], Graphics3D [{stick2 , Dtri[Γ3[0]], Dtri[Γ3[.5]]}],
   Graphics3D [{stick2 , Dtri[Γ3[0]], Dtri[Γ3[1]]}]},
  {Graphics3D [{stick2 , Dtri[Γ3[0]], Dtri[Γ3[1.5]]}],
   Graphics3D [{stick2 , Dtri[Γ3[0]], Dtri[Γ3[2]]}]}}]
```

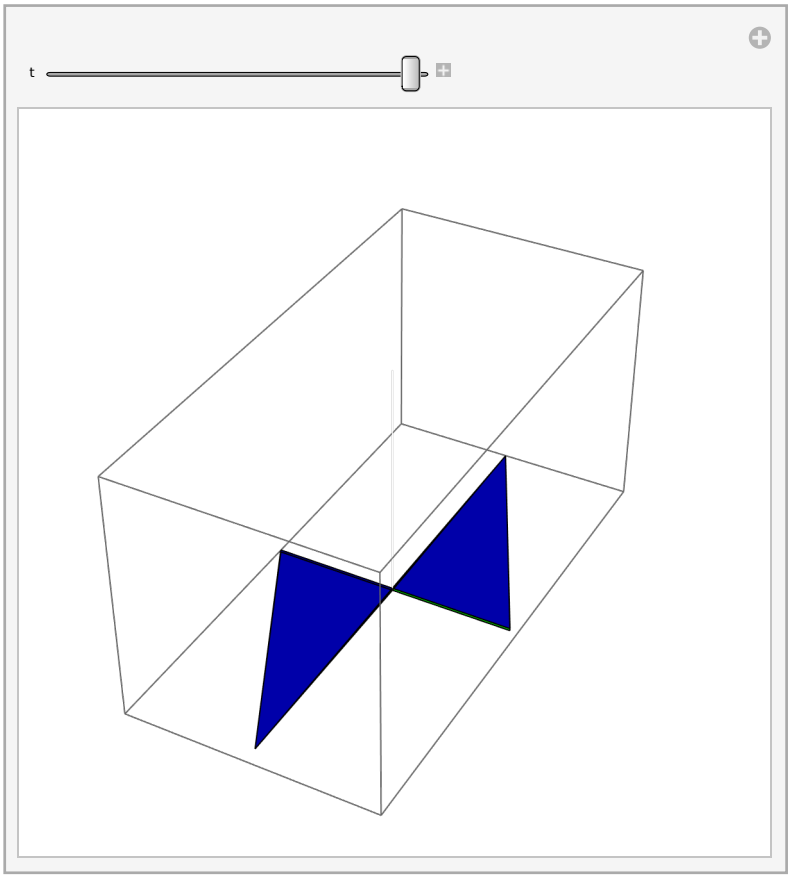
Out[ ]=



For those reading the Wolfram notebook in Mathematica

In[ ]:=
```
Manipulate[Graphics3D[{stick2, Dtri[Γ3[0]], Dtri[Γ3[t]]}], {t, 0, 2}]
```

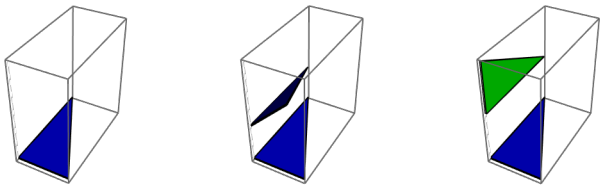
Out[ ]=



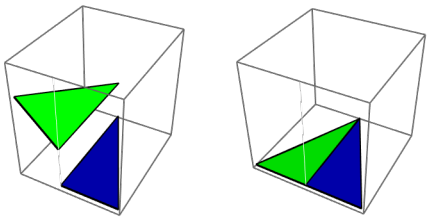

Since this is an even transformation the original blue side is always up. In the case of a reflection we see

In[ ]:=
```
Γ1[t_] := TranslationTransform[G[t]]@* RotationTransform[-t * Pi / 2, {0, 1, 0}]
```

In[ ]:=
```
GraphicsGrid [
 {{Graphics3D [{stick2 , Dtri[Γ1[0]]}], Graphics3D [{stick2 , Dtri[Γ1[0]], Dtri[Γ1[.5]]}],
    Graphics3D [{stick2 , Dtri[Γ1[0]], Dtri[Γ1[1]]}]},
  {Graphics3D [{stick2 , Dtri[Γ1[0]], Dtri[Γ1[1.5]]}],
    Graphics3D [{stick2 , Dtri[Γ1[0]], Dtri[Γ1[2]]}]}}]
```



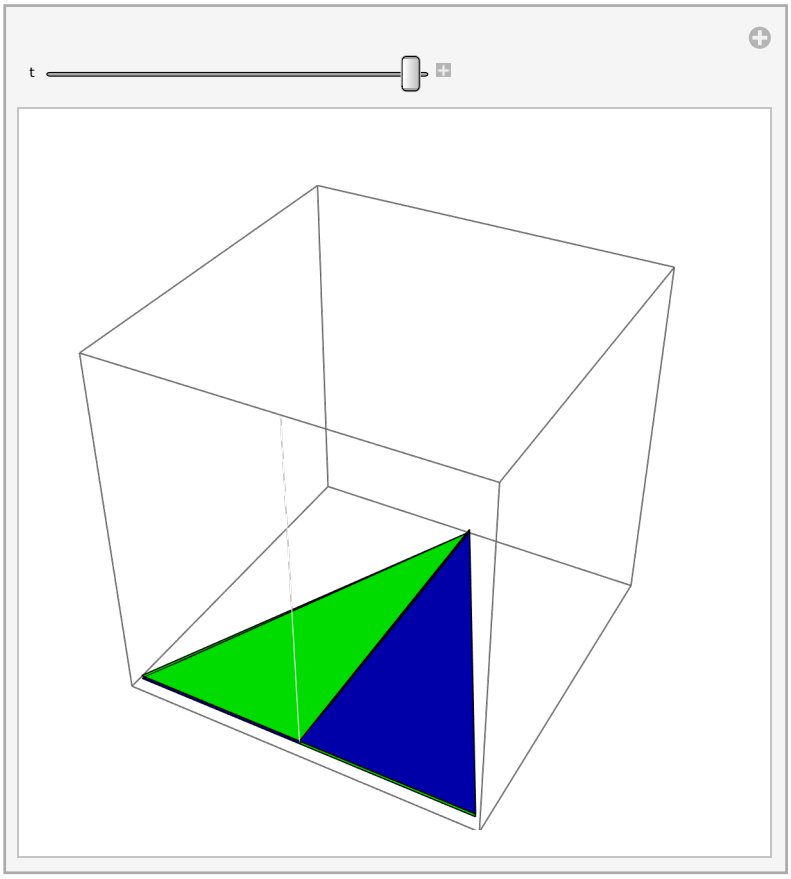
Out[ ]=



Note that this is a rotation in 3D but the result is a reflection in 2D. Therefore we see the other side up. or, in Mathematica

In[ ]:=
```
Manipulate[Graphics3D[{stick2, Dtri[Γ1[0]], Dtri[Γ1[t]]}], {t, 0, 2}]
```

Out[ ]=



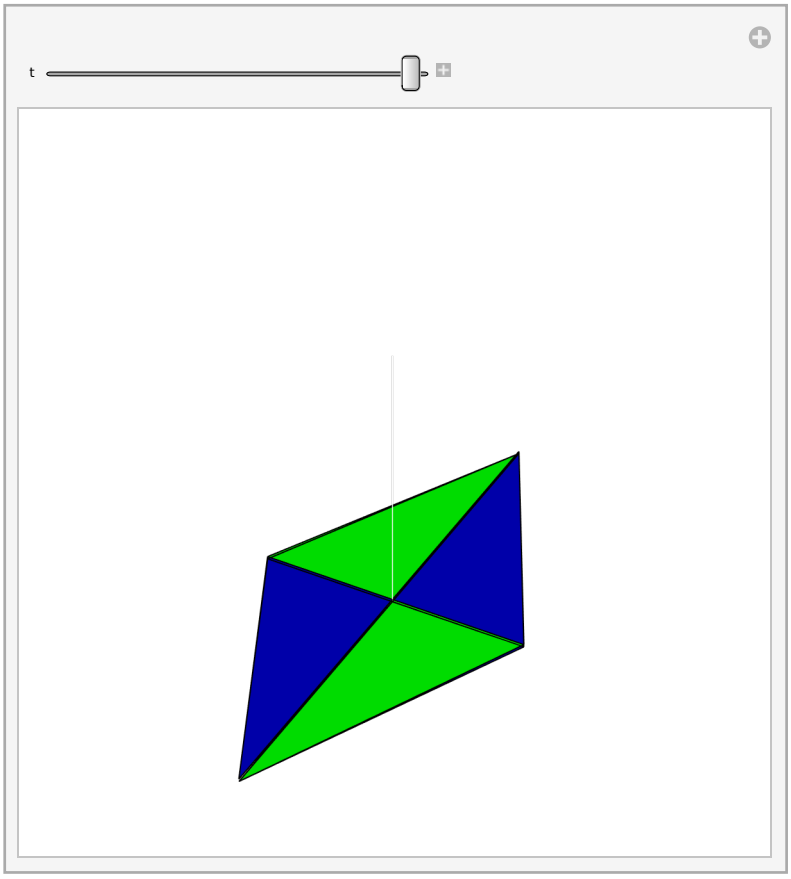

We can put these together along with a third transformation

In[ ]:=
```
Γ2[t_] := TranslationTransform[G[t]]@* RotationTransform[t Pi / 2, {1, 0, 0}]
```

In[ ]:=
```
Γ[t_] := Piecewise[{{Γ1[t], 0 ≤ t ≤ 2}, {Γ1[2], 2 < t < 3},
    {Γ2[t - 3], 3 ≤ t < 5}, {Γ2[2], 5 ≤ t < 6}, {Γ3[t - 6], 6 ≤ t < 8}}, Γ3[2]]
```

In[ ]:=
```
Manipulate[If[t < 0, Graphics3D[{stick2, Dtri[Γ[0]]}],
  If[0 ≤ t < 2, Graphics3D[{stick2, {Dtri[Γ[0]], Dtri[Γ[t]]}}],
   If[2 ≤ t < 3, Graphics3D[{stick2, Dtri[Γ[0]], Dtri[Γ[t]]}],
    If[3 ≤ t ≤ 5, Graphics3D[{stick2, Dtri[Γ[0]], Dtri[Γ[2]], Dtri[Γ[t]]}],
     If[5 < t < 6, Graphics3D[{stick2, Dtri[Γ[0]], Dtri[Γ[2]], Dtri[Γ[5]]}],
      If[6 ≤ t, Graphics3D[{stick2, Dtri[Γ[0]], Dtri[Γ[2]], Dtri[Γ[5]], Dtri[Γ[t]]},
       Boxed → False]]]]], {t, -2, 8}]
```

Out[ ]=




Those reading this in pdf format may see the moving graphic on YouTube
https://youtube.com/shorts/WPQYpfFQddI?si=l2O08cC6J0U56T9t