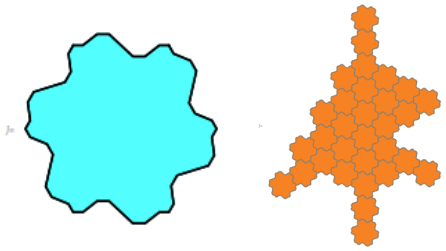
Yale's C6, one rotation of order 6

We could proceed as in C3 starting with the plane version in Chapter 4 and making it into a prism. For C6 we had the shape and tessellation



Already for C3 there was still the complication that simply adding a third dimension we had the problem of the horizontal reflection. We solved it there by using a colored shape such as in C4. But then we could have gotten a simpler figure just starting with a triangular prism and appropriate coloring. That is my approach here, coloring a hexagonal prism. This is the reason for the generators below.

But first we look at the group. We would like to have

tasC6 := ‹| 1 → τh1, 2 → τh2, 3 → τh3, 4 → σh6 |›

where

In[•]:=
```
τh1 := τb[{Sqrt[3.], 0, 0}]
τh2 := τb[{Sqrt[3] / 2, 1.5, 0}]
τh3 := τb[{0, 0, 1}]
σh6 := rotationTF3D[Pi / 3, {{0, 0, 0}, {0, 0, 1}}]
```

But we need inverses . Now σh6 has finite order, but it's inverse would not appear until level 5. As in the plane case we can simplify things by noting that

In[•]:=
```
σh6@*σh6@*σh6
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

which is our ηz.

In[•]:=
```
ηz@*τh1
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} -1. & 0. & 0. & -1.73205 \\ 0. & -1. & 0. & 0. \\ 0. & 0. & 1. & 0. \\ \hline 0. & 0. & 0. & 1. \end{array}\right)\right]$$

In[•]:= **ηz@* τh2**

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} -1. & 0. & 0. & -0.866025 \\ 0. & -1. & 0. & -1.5 \\ 0. & 0. & 1. & 0. \\ \hline 0. & 0. & 0. & 1. \end{array}\right)\right]$$

are the inverses for τh1 and τh2

ηz would not appear in our group until level 3. But if we add it to our generators then we have inverses for τh1 and τh2 in level 2. Also we have an inverse to σh6 in level 3. So we will use the generating set

In[•]:= **tasC6 := <|1 → τh1, 2 → τh2, 3 → τh3, 4 → ηz, 5 → σh6, 6 → τb[{0, 0, -1}]|>**

In[•]:= **tp6 = RandomReal[{-2, 2}, 3]**

Out[•]=

```
{1.68088, -1.70268, 1.21505}
```

In[•]:= **C6[3] = transGroup[tasC6, tp6, 3]**

» number of group elements calculated 93

Out[•]=

```
{{1}, {2}, {3}, {4}, {5}, {6}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6},
 {2, 2}, {2, 3}, {2, 4}, {2, 5}, {2, 6}, {3, 3}, {3, 4}, {3, 5}, {3, 6}, {4, 1},
 {4, 2}, {4, 5}, {4, 6}, {5, 1}, {5, 5}, {5, 6}, {6, 6}, {1, 1, 1}, {1, 1, 2},
 {1, 1, 3}, {1, 1, 4}, {1, 1, 5}, {1, 1, 6}, {1, 2, 2}, {1, 2, 3}, {1, 2, 4},
 {1, 2, 5}, {1, 2, 6}, {1, 3, 3}, {1, 3, 4}, {1, 3, 5}, {1, 4, 2}, {1, 4, 5}, {1, 4, 6},
 {1, 5, 1}, {1, 5, 5}, {1, 5, 6}, {1, 6, 6}, {2, 2, 2}, {2, 2, 3}, {2, 2, 4}, {2, 2, 5},
 {2, 2, 6}, {2, 3, 3}, {2, 3, 4}, {2, 3, 5}, {2, 4, 1}, {2, 4, 5}, {2, 4, 6}, {2, 5, 5},
 {2, 5, 6}, {2, 6, 6}, {3, 3, 3}, {3, 3, 4}, {3, 3, 5}, {3, 4, 1}, {3, 4, 2}, {3, 4, 5},
 {3, 5, 1}, {3, 5, 5}, {4, 1, 1}, {4, 1, 2}, {4, 1, 4}, {4, 1, 5}, {4, 1, 6}, {4, 2, 2},
 {4, 2, 4}, {4, 2, 5}, {4, 2, 6}, {4, 5, 1}, {4, 5, 5}, {4, 5, 6}, {4, 6, 6}, {5, 1, 1},
 {5, 1, 4}, {5, 1, 5}, {5, 1, 6}, {5, 5, 1}, {5, 5, 6}, {5, 6, 6}, {6, 6, 6}}
```

In[•]:= **GroupFindKey[ι, C6[3], tasC6, tp6]**

Out[•]=

```
{3, 6}
```

In[•]:= **GroupFindKey[InverseTF[τh1], C6[3], tasC6, tp6]**

Out[•]=

```
{4, 1, 4}
```

In[•]:= **GroupFindKey[InverseTF[τh2], C6[3], tasC6, tp6]**

Out[•]=

```
{4, 2, 4}
```

In[•]:= **GroupFindKey[InverseTF[σh6], C6[3], tasC6, tp6]**

Out[•]=

```
{4, 5, 5}
```

So we have all our inverses in C6[3] . Looking at the classifications we have

In[•]:= **classCountG[C6[3], tasC6]**

```
ι  1
T  31
H  13
SD2  12
SD3  4
SD6  10
R3  12
R6  10
```

Note that we already must have, besides our half turn ηz, also other rotations of order 2 and 3.

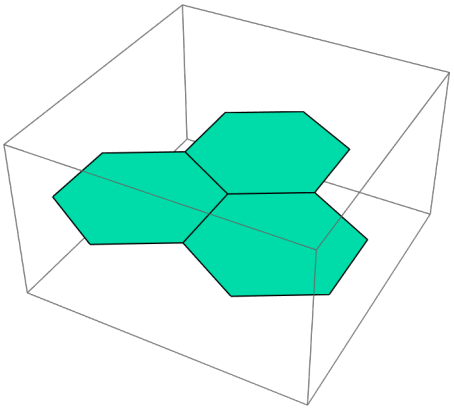
We begin our construction

In[•]:=
```
Dhex[α_] :=
 α /@ {{0, 1, 0}, {-Sqrt[3]/2, 1/2, 0}, {-Sqrt[3]/2, -1/2, 0}, {0, -1, 0}, {Sqrt[3]/2, -1/2, 0}, {Sqrt[3]/2, 1/2, 0}}
```

A view of this is

```
Graphics3D[{Cyan, Polygon[Dhex[ι]], Polygon[Dhex[τh1]], Polygon[Dhex[τh2]]}]
```

Out[•]=

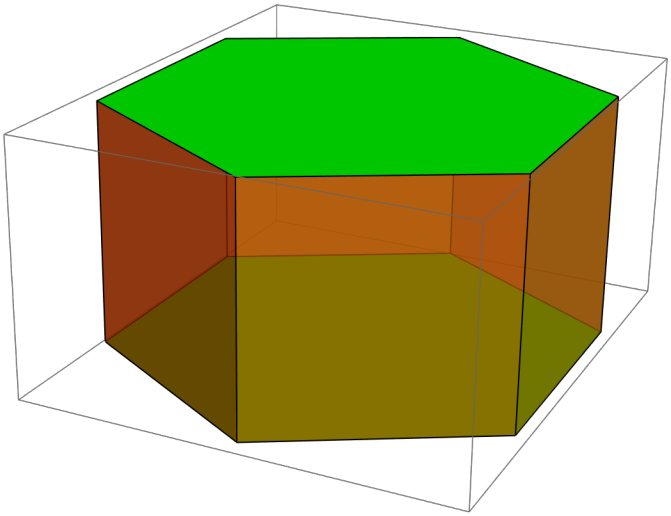


Here is our hexagonal prism.

In[•]:=
```
panel1 := {{0, 1, 0}, {-Sqrt[3]/2, 1/2, 0}, {-Sqrt[3]/2, 1/2, 1}, {0, 1, 1}}
```

In[•]:=
```
Graphics3D[{{Green, Polygon[Dhex[ι]], Polygon[Dhex[τh3]]},
  {Orange, Opacity[.75], Polygon[panel1], Polygon[σh6@panel1], Polygon[σh6@*σh6@panel1],
   Polygon[ηz@panel1], Polygon[σh6@*ηz@panel1], Polygon[σh6@*σh6@*ηz@panel1]}}]
```

Out[•]=



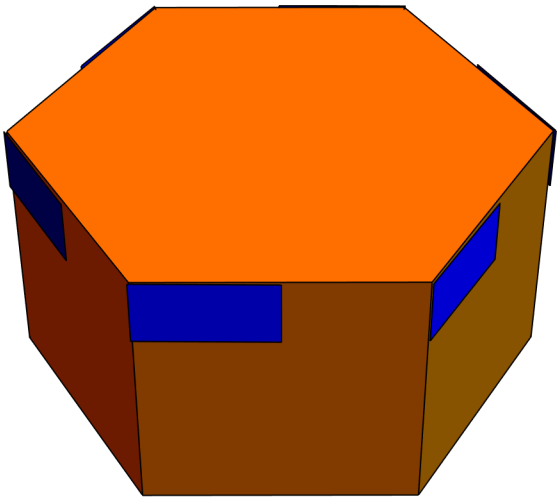
As it stands, there are many reflections also that leave this fixed . A little paint will cure this .

```
blue = {{0, 1.01, 1}, {-Sqrt[3] / 4 - .01, 3 / 4 + .01, 1},
  {-Sqrt[3] / 4 - .01, 3 / 4 + .01, .75}, {0, 1.01, .75}}
```

In[•]:=
```
DHex[α_] :=
 {{Orange, Polyhedron[α /@ {panel1, σh6@panel1, σh6@*σh6@panel1, ηz@panel1, σh6@*ηz@
      panel1, σh6@*σh6@*ηz@panel1, {{0, 1, 0}, {-Sqrt[3]/2, 1/2, 0}, {-Sqrt[3]/2, -1/2, 0},
      {0, -1, 0}, {Sqrt[3]/2, -1/2, 0}, {Sqrt[3]/2, 1/2, 0}}, {{0, 1, 1}, {-Sqrt[3]/2, 1/2, 1},
      {-Sqrt[3]/2, -1/2, 1}, {0, -1, 1}, {Sqrt[3]/2, -1/2, 1}, {Sqrt[3]/2, 1/2, 1}}}]],
  {Blue, Polygon[α@blue], Polygon[α@*σh6@blue], Polygon[α@*σh6@*σh6@blue],
   Polygon[α@*ηz@blue], Polygon[α@*σh6@*ηz@blue],
   Polygon[α@*σh6@*σh6@*ηz@blue]}}
```

In[•]:=
```
Graphics3D[DHex[ι], Boxed → False]
```

Out[•]=



We can now build crystals using our group

In[•]:=
```
K = {{3, 6}, {1}, {2}, {1, 1}, {1, 2},
    {2, 2}, {3}, {3, 1}, {3, 2}, {3, 1, 1}, {3, 1, 2}, {3, 2, 2}};
```

In[•]:=
```
Graphics3D[Table[DHex[TasTF[g, tasC6]], {g, K}], Boxed → False]
```

Out[•]=

