Yale's C4

We take the three translations of Barry's Box $\tau bx, \tau by, \tau bz$ and add the rotation $\sigma 4x$. Note that

In[•]:=
```
σ4x@*σ4x
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

is the same as

In[•]:=
```
ηx
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

we know

In[•]:=
```
ηx@*τby@*ηx
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

In[•]:=
```
ηx@*τbz@*ηx
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

so τby and τbz will already have inverses in level 5 aBut τbx will not have an so we also need that as generator. So our group GC4 is generated by

In[•]:=
```
tasC4 := <|1 → τbx, 2 → τby, 3 → τbz, 4 → σ4x, 5 → τb[{-1, 0, 0}]|>
```

In[•]:=
```
tc4 = RandomReal[{-2, 2}, 3]
```

Out[•]=
```
{-1.65236, 1.17689, 0.445332}
```

In[•]:= **GC4[3] = transGroup[tasC4, tc5, 3]**

» number of group elements calculated 62

Out[•]=

```
{{1}, {2}, {3}, {4}, {5}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 2}, {2, 3},
 {2, 4}, {2, 5}, {3, 3}, {3, 4}, {3, 5}, {4, 2}, {4, 4}, {4, 5}, {5, 5}, {1, 1, 1},
 {1, 1, 2}, {1, 1, 3}, {1, 1, 4}, {1, 1, 5}, {1, 2, 2}, {1, 2, 3}, {1, 2, 4}, {1, 2, 5},
 {1, 3, 3}, {1, 3, 4}, {1, 3, 5}, {1, 4, 2}, {1, 4, 4}, {1, 4, 5}, {1, 5, 5}, {2, 2, 2},
 {2, 2, 3}, {2, 2, 4}, {2, 2, 5}, {2, 3, 3}, {2, 3, 4}, {2, 3, 5}, {2, 4, 2}, {2, 4, 4},
 {2, 4, 5}, {2, 5, 5}, {3, 3, 3}, {3, 3, 4}, {3, 3, 5}, {3, 4, 4}, {3, 4, 5}, {3, 5, 5},
 {4, 2, 2}, {4, 2, 4}, {4, 2, 5}, {4, 4, 2}, {4, 4, 4}, {4, 4, 5}, {4, 5, 5}, {5, 5, 5}}
```

In[•]:= **GroupFindKey[ι, GC4[3], tasC4, tc3]**

Out[•]=

```
{1, 5, 5}
```

In[•]:= **GroupFindKey[InverseTF[τby], GC4[3], tasC4, tc3]**

Out[•]=

```
Fail
```

but here it is

In[•]:= **TasTF[{4, 4, 2, 4, 4}, tasC4]**

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

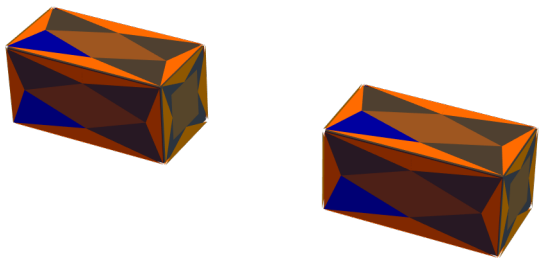
In[•]:= **classCountG[GC4[3], tasC4]**

```
ι  1
T  33
H  5
SD2  2
SD4  11
R4  10
```

To show that this is discrete we use the following modified version of my box

In[•]:= **Graphics3D[{DBbox3[ι], DBbox3[τbx@*τbx@*ηx]}, Boxed → False]**

Out[•]=

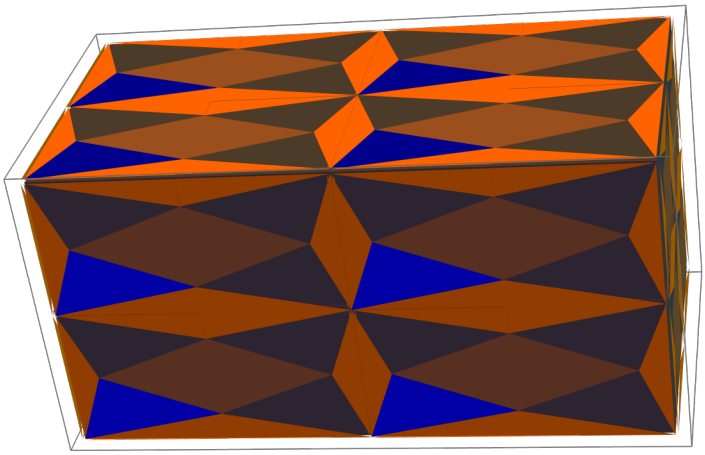


This shows the bottom left hand corner of each side is marked blue . So **σ4x** leaves the colored box unchanged but any other isometry not in our group will not. So this is the transformation group of DBbox3 and hence discrete. We can build crystals using tasC4 as in earlier groups.

In[•]:=
```
K = {{1, 5, 5}, {1}, {2}, {1, 2}, {3}, {3, 1}, {3, 2}, {3, 1, 2}};
```

In[•]:=
```
Graphics3D[Table[DBbox3[TasTF[k, tasC4]], {k, K}]]
```

Out[•]=



This colored crystal has 4-fold rotation along the large axis.