Yale' s C3

Here we have only the translations and the rotation of order 3 about the vertical line through the origin . Specifically

In[•]:=
```
σc3 = rotationTF3D[2 Pi / 3, {{0, 0, 0}, {0, 0, 1}}]
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

Here we use the triangular shape in Section 4.5.8 which is has the (planar) rotation of order 3 as its only non-identity symmetry. In 2 dimensions it is defined in the closed cell below for compactness of display.

We let the 2 dimensional identity transformation be

In[•]:=
```
ι2 := TranslationTransform[{0, 0}]
```

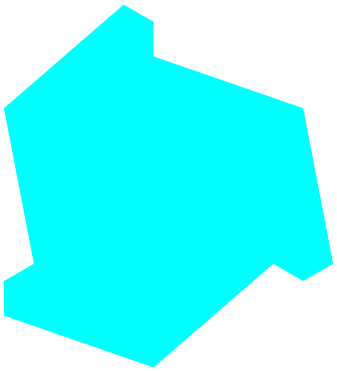
Now here is the definition

In[•]:=
```
D32D[ι2][[1]]
```

Out[•]=
```
{{0., -0.57735}, {0.4, -0.23094}, {0.5, -0.288675}, {0.5, -0.288675}, {0.6, -0.23094},
 {0.5, 0.288675}, {0.5, 0.288675}, {0., 0.46188}, {0., 0.57735}, {0., 0.57735},
 {-0.1, 0.635085}, {-0.5, 0.288675}, {-0.5, 0.288675}, {-0.4, -0.23094},
 {-0.5, -0.288675}, {-0.5, -0.288675}, {-0.5, -0.404145}, {0., -0.57735}}
```

In[•]:=
```
Graphics[{Cyan, D32D[ι2]}, ImageSize → Small]
```

Out[•]=



The two dimensional version of our rotation is

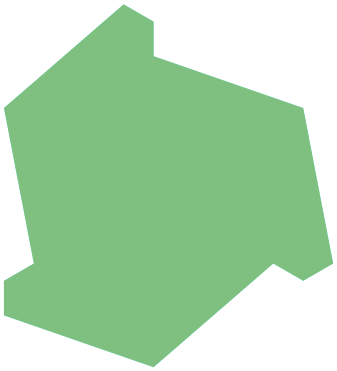
```
In[•]:= σ32D = TransformationFunction[{{-.5, -Sqrt[3] / 2, 0}, {Sqrt[3] / 2, -.5, 0}, {0, 0, 1}}]
```

Out[•]=

$$\text{TransformationFunction}\left[\left(\begin{array}{cc|c} -0.5 & -0.866025 & 0. \\ 0.866025 & -0.5 & 0. \\ \hline 0. & 0. & 1. \end{array}\right)\right]$$

```
In[•]:= Graphics[{{Cyan, D32D[ι2]}, {Orange, Opacity[.5], D32D[σ32D]}}, ImageSize → Small]
```

Out[•]=



Shows σ32D is a symmetry of this triangular shape . To make this into a tesselation we used translations

```
In[•]:= τh := TranslationTransform[{1, 0}]
```

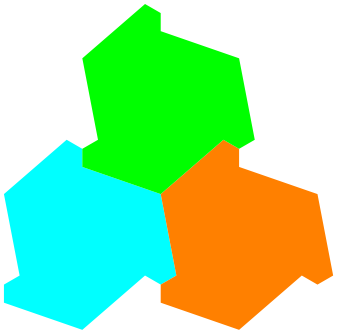
```
In[•]:= τd = TranslationTransform[{-.5, Sqrt[3] / 2}]
```

Out[•]=

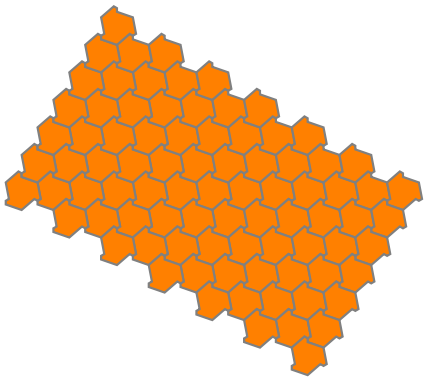
$$\text{TransformationFunction}\left[\left(\begin{array}{cc|c} 1. & 0. & -0.5 \\ 0. & 1. & 0.866025 \\ \hline 0. & 0. & 1. \end{array}\right)\right]$$

```
In[•]:= Graphics[{{Cyan, D32D[ι2]}, {Orange, D32D[τh]}, {Green, D32D[τd@* τh]}}, ImageSize → Small]
```

Out[•]=



In Chapter 4.8 we get a bigger picture of this tessellation.



So here we will get a 3 dimensional lattice by making this 2D shape into a prism.

In additon to σc3 above we now start with

In[164]:=
```
τc31 := τb[{1, 0, 0}]
```

Given these two elements of our group then we must have the translation

In[165]:=
```
τc32 = σc3@* τ1@* σc3@* σc3
```

Out[165]=

$$\text{TransformationFunction}\left[\left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\right]$$

Which we will use instead of `τb[{0, 1, 0}]`

We can add

In[166]:=
```
τc33 := τb[{0, 0, 1}]
```

which is independent from the two plane translations .

We get a group from

In[167]:=
```
tasC3 := <|1 → τc31, 2 → τc32, 3 → τc33,
  4 → τb[{-1, 0, 0}], 5 → InverseTF[τc32], 6 → τb[{0, 0, -1}], 7 → σc3|>
```

Take for now our test point

In[•]:=
```
tc3 = RandomReal[{-2, 2}, 3]
```

Out[•]=
```
{1.6694, -1.31702, -0.756547}
```

In[•]:=
```
classTF3D[τc32]
```

Out[•]=

T

In[•]:=
```
GC3[3] = transGroup[tasC3, tc3, 3]
```

» number of group elements calculated 107

Out[•]=
```
{{1}, {2}, {3}, {4}, {5}, {6}, {7}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6},
 {1, 7}, {2, 2}, {2, 3}, {2, 4}, {2, 6}, {2, 7}, {3, 3}, {3, 4}, {3, 5}, {3, 7}, {4, 4},
 {4, 5}, {4, 6}, {4, 7}, {5, 5}, {5, 6}, {5, 7}, {6, 6}, {6, 7}, {7, 1}, {7, 4}, {7, 7},
 {1, 1, 1}, {1, 1, 2}, {1, 1, 3}, {1, 1, 5}, {1, 1, 6}, {1, 1, 7}, {1, 2, 2}, {1, 2, 3},
 {1, 2, 6}, {1, 3, 3}, {1, 3, 5}, {1, 3, 7}, {1, 5, 5}, {1, 5, 6}, {1, 5, 7}, {1, 6, 6},
 {1, 6, 7}, {1, 7, 4}, {1, 7, 7}, {2, 2, 2}, {2, 2, 3}, {2, 2, 4}, {2, 2, 6}, {2, 2, 7},
 {2, 3, 3}, {2, 3, 4}, {2, 3, 7}, {2, 4, 4}, {2, 4, 6}, {2, 4, 7}, {2, 6, 6}, {2, 6, 7},
 {2, 7, 4}, {2, 7, 7}, {3, 3, 3}, {3, 3, 4}, {3, 3, 5}, {3, 3, 7}, {3, 4, 4}, {3, 4, 5},
 {3, 4, 7}, {3, 5, 5}, {3, 5, 7}, {3, 7, 1}, {3, 7, 4}, {3, 7, 7}, {4, 4, 4}, {4, 4, 5},
 {4, 4, 6}, {4, 4, 7}, {4, 5, 5}, {4, 5, 6}, {4, 6, 6}, {4, 6, 7}, {4, 7, 1}, {4, 7, 7},
 {5, 5, 5}, {5, 5, 6}, {5, 5, 7}, {5, 6, 6}, {5, 6, 7}, {5, 7, 1}, {5, 7, 7}, {6, 6, 6},
 {6, 6, 7}, {6, 7, 1}, {6, 7, 4}, {6, 7, 7}, {7, 1, 1}, {7, 1, 7}, {7, 4, 4}, {7, 4, 7}}
```

In[•]:=
```
GroupFindKey[ι, GC3[3], tasC3, tc3]
```

Out[•]=
```
{1, 4}
```

We have in this group

In[•]:=
```
classCountG[GC3[3], tasC3]
```

```
ι  1

T  62

SD3  18

R3  26
```

To show this is discrete we can use these transformation functions to construct a3 dimensional Prism. First we add a 0 to get our floor

In[•]:=
```
P3
```

Out[•]=
```
{{0., -0.57735, 0}, {0.4, -0.23094, 0}, {0.5, -0.288675, 0},
 {0.5, -0.288675, 0}, {0.6, -0.23094, 0}, {0.5, 0.288675, 0}, {0., 0.46188, 0},
 {0., 0.57735, 0}, {-0.1, 0.635085, 0}, {-0.5, 0.288675, 0}, {-0.4, -0.23094, 0},
 {-0.5, -0.288675, 0}, {-0.5, -0.288675, 0}, {-0.5, -0.404145, 0}, {0., -0.57735, 0}}
```

and our ceiling is

In[•]:=
```
P3T = τc33 /@ P3
```

Out[•]=
```
{{0., -0.57735, 1.}, {0.4, -0.23094, 1.}, {0.5, -0.288675, 1.},
 {0.5, -0.288675, 1.}, {0.6, -0.23094, 1.}, {0.5, 0.288675, 1.}, {0., 0.46188, 1.},
 {0., 0.57735, 1.}, {-0.1, 0.635085, 1.}, {-0.5, 0.288675, 1.}, {-0.4, -0.23094, 1.},
 {-0.5, -0.288675, 1.}, {-0.5, -0.288675, 1.}, {-0.5, -0.404145, 1.}, {0., -0.57735, 1.}}
```

We construct 4 panels

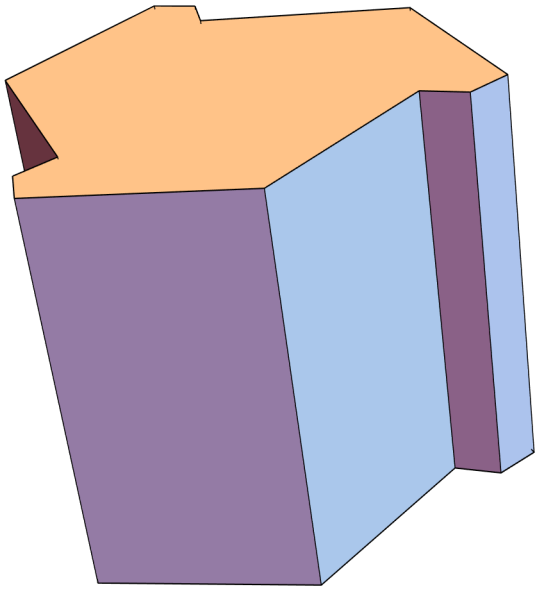
```
In[•]:= panel1 = {P3[[1]], P3T[[1]], P3T[[2]], P3[[2]]}
Out[•]=
{{0., -0.57735, 0}, {0., -0.57735, 1.}, {0.4, -0.23094, 1.}, {0.4, -0.23094, 0}}

In[•]:= panel2 = {P3[[2]], P3[[3]], P3T[[3]], P3T[[2]]}
Out[•]=
{{0.4, -0.23094, 0}, {0.5, -0.288675, 0}, {0.5, -0.288675, 1.}, {0.4, -0.23094, 1.}}

In[•]:= panel3 = {P3[[4]], P3[[5]], P3T[[5]], P3T[[4]]}
Out[•]=
{{0.5, -0.288675, 0}, {0.6, -0.23094, 0}, {0.6, -0.23094, 1.}, {0.5, -0.288675, 1.}}

In[•]:= panel4 = {P3[[5]], P3[[6]], P3T[[6]], P3T[[5]]}
Out[•]=
{{0.6, -0.23094, 0}, {0.5, 0.288675, 0}, {0.5, 0.288675, 1.}, {0.6, -0.23094, 1.}}

In[•]:= Graphics3D[Polyhedron[{P3, P3T, panel1, σc3@panel1,
    σc3@* σc3@panel1, panel2, σc3@panel2, σc3@* σc3@panel2, panel3, σc3@panel3,
    σc3@* σc3@panel3, panel4, σc3@panel4, σc3@* σc3@panel4}], Boxed → False]
Out[•]=
```



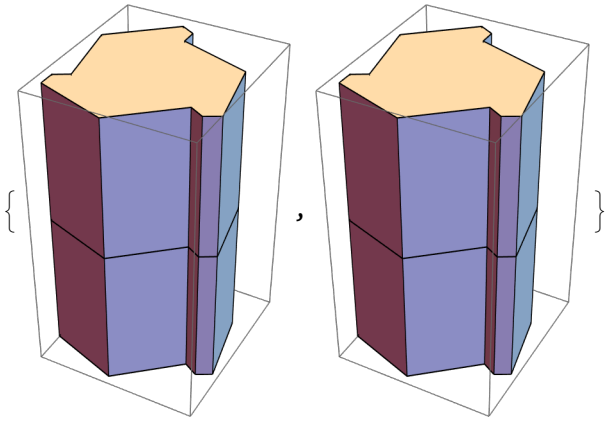
So we have the dynamic

```
In[•]:= DC3[α_] := Polyhedron[
  α /@ {P3, P3T, panel1, σc3@panel1, σc3@* σc3@panel1, panel2, σc3@panel2, σc3@* σc3@panel2,
    panel3, σc3@panel3, σc3@* σc3@panel3, panel4, σc3@panel4, σc3@* σc3@panel4}]
```

We do have a problem that GC3 is not the full group of isometries of this prism, the reflection $\rho 3$ is also an isometry.

```
In[•]:= {Graphics3D[{DC3[ι], DC3[τb[{0, 0, -1}]]}, ImageSize → Small],
         Graphics3D[{DC3[ι], DC3[ρ3]}, ImageSize → Small]}
```

Out[•]=



Visually and geometrically these are the same . We need to orient our prism leaving the transformations in tasC3 the same. We bring in some size transformations which are not isometries or even similarities as they act only on the 3rd variable.

```
In[•]:= size1 := TransformationFunction[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, .3333, 0}, {0, 0, 0, 1}}]
        size2 := TransformationFunction[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, .6667, 0}, {0, 0, 0, 1}}]
```

and another translation

```
up3z := TranslationTransform[{0, 0, .33333}]
```

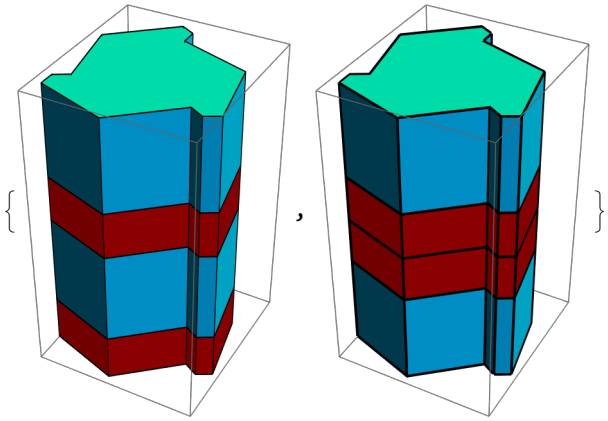
and a new dynamic prism

```
In[•]:= DT3[α_] :=
         {{Red, Polyhedron[α@*size1 /@ {P3, P3T, panel1, σc3@panel1, σc3@*σc3@panel1, panel2,
               σc3@panel2, σc3@*σc3@panel2, panel3, σc3@panel3,
               σc3@*σc3@panel3, panel4, σc3@panel4, σc3@*σc3@panel4}]},
          {Cyan, Polyhedron[α@*up3@*size2 /@ {P3, P3T, panel1, σc3@panel1,
               σc3@*σc3@panel1, panel2, σc3@panel2, σc3@*σc3@panel2, panel3,
               σc3@panel3, σc3@*σc3@panel3, panel4, σc3@panel4, σc3@*σc3@panel4}]}}
```

Now

In[•]:=
```
{Graphics3D[{DT3[ι], DT3[τb[{0, 0, -1}]]}, ImageSize → Small],
 Graphics3D[{DT3[ι], DT3[ρ3]}, ImageSize → Small]}
```

Out[•]=



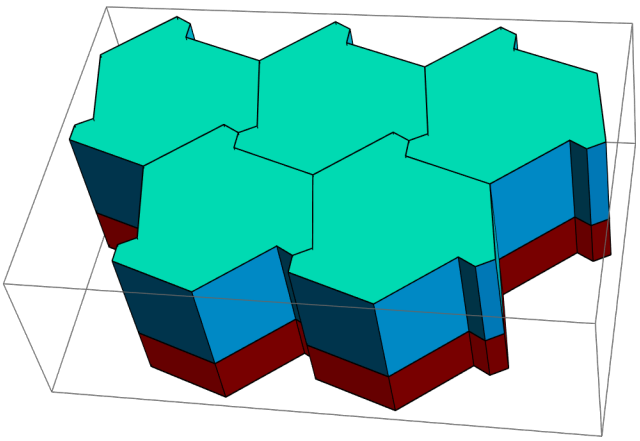
Give different colored prisms .

We can draw by

In[•]:=
```
K = {{1, 4}, {1}, {4}, {5}, {2, 7}};
```

In[•]:=
```
Graphics3D[Table[DT3[TasTF[k, tasC3]], {k, K}]]
```

Out[•]=



These all fit nicely .