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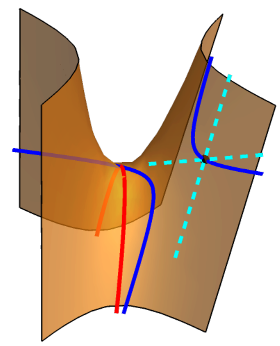

Front Graphic

This graphic depicts the saddle surface, sometimes misnamed as a *hyperbolic paraboloid* as it is a *hyperboloid,* given by the equation $z = x\,y$. The intersection of this surface with the plane $x + y = 0$ is the parabola shown in red. The saddle surface has many projective symmetries, that is, invertible projective linear transformations that take the saddle surface onto itself. One of these can be given by the 4×4 matrix for homogeneous coordinates {x,y,z,w}.

```
In[ ]:= sr3 =
```

$$\begin{pmatrix} 4.297877056362575` & 1.1516126864206027` & 2.224744871391589` & 2.224744871391589` \\ 0.11633650601052004` & 0.4341737512063021` & 0.22474487139158894` & 0.22474487139158894` \\ 0.7071067811865475` & 0.7071067811865475` & 1.3660254037844386` & 0.3660254037844386` \\ 0.7071067811865475` & 0.7071067811865475` & 0.3660254037844386` & 1.3660254037844386` \end{pmatrix};$$

This is given by its machine number equivalent here but in fact is exact, expressible by complicated entries using $\sqrt{2}$, $\sqrt{3}$. You may, for example, recognize the first two entries in the bottom two rows.

The blue curve is the image of the red parabola under this symmetry. The black point on this curve is the image of the invisible, (AKA infinite, ideal) point of the parabola. The two dashed lines form the image of the invisible curve. Specifically if one writes the homogeneous equation of this hyperboloid $z\,w - x\,y = 0$ the preimage of these lines form the intersection of this surface by the plane $w = 0$. Thus from this one picture we can see what the entire hyperboloid looks like. A major takeaway from this picture is that this saddle surface (which could be called a parabolic hyperboloid) is tangent to the plane at infinity. In fact, in affine space this is what distinguishes the parabolic hyperboloid from other hyperboloids.

For more information see section 2.3.10 in the preview to Chapter 2 of my Surface Story Book

https://barryhdayton.space/SurfaceBook/Chap2Sec3preview.pdf